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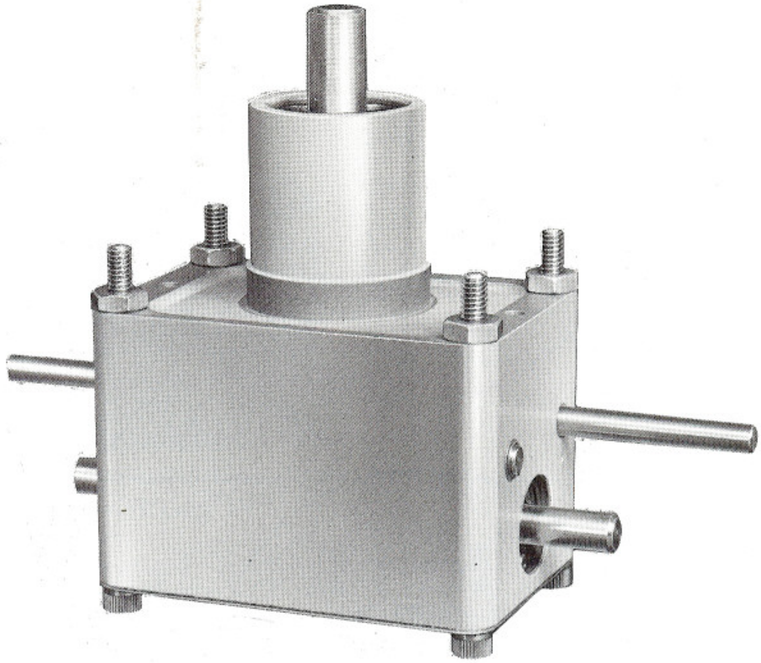


LIBRASCOPE

BALL-AND-DISC INTEGRATOR

DESCRIPTION

Librascope's high-torque Ball-and-Disc Integrator features an adjustable microbearing ball carriage whose rolling action minimizes the sliding friction inherent in conventional sleeve-type carriages. This new design concept greatly increases the integrator's useful service life. A technique is provided for constraining the balls more accurately than is possible with nonadjustable types, which rely on machining to maintain tolerances.

The Ball-and-Disc Integrator is a precision mechanical integrating element containing a working mechanism of four primary parts: (1) a rotating tungsten-carbide disc, for entering the time factor; (2) a pair of precision-steel balls retained in (3) the microbearing ball carriage and integral ball-carriage shaft, for entering the variable input function; and (4) a precision-ground, rotary-output cylinder. Spring loading facilitates transfer of disc rotation through the balls to the output cylinder. The output cylinder's rotational speed is proportional to the product of the disc velocity and the displacement of the balls from the disc center.

The operation of the Librascope integrator may be slowed or stopped in mid-solution without affecting accuracy.

As a computing element, a variable-speed changer, or a control device, this new integrator provides the engineer with a versatile, low-cost unit featuring greater reliability, accuracy, and ruggedness than any other integrator presently available.

FEATURES

Adjustable microbearing ball-carriage · Integrand input for ball-carriage positioning may be shaft, rack, or lead screw · High output-torque capabilities · Low input-force requirements · Accuracy maintained with reverse inputs · Life expectancy: 10,000 hours.

APPLICATIONS

Designed to serve varied computing and control needs, the integrator performs three fundamental functions: integration, differentiation, and speed control. Thus, the device can operate in an almost limitless number of commercial and military applications including weapon fire control, navigation, and process control systems; machine-tool positioning, and servo systems. Since it requires no electrical power, it is particularly suitable for hazardous environments, remote locations, portable instruments, pneumatic and pipeline control systems.

Ball-and-Disc Integrator

FIGURE 1

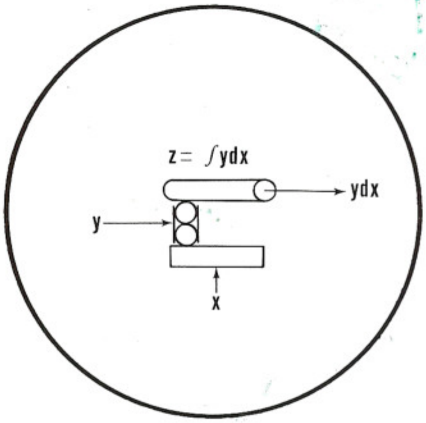


FIGURE 2

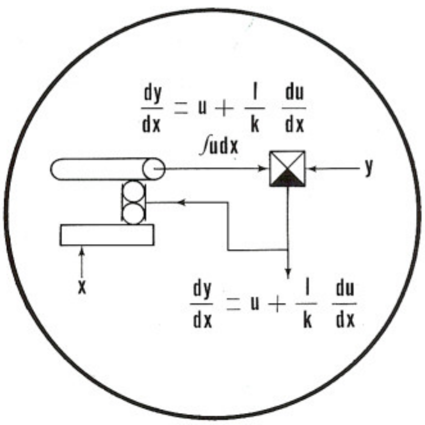
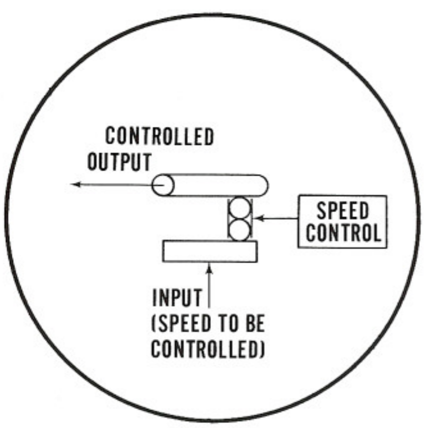


FIGURE 3



INTEGRATION

Figure 1 schematically represents the solution of the equation $Z = \frac{1}{r}\int ydx$. The ball-carriage position is the function $y = f(x)$. The disc's angular rotation represents the quantity x; and $\frac{1}{r}$, the integrator constant where $r =$ the output cylinder's radius in inches. If two variable inputs are required, both disc speed and ball-carriage position may be varied.

DIFFERENTIATION

Figure 2 illustrates the method used to obtain mathematical differentiation. By coupling a mechanical differential to the integrator, as shown, derivative $\frac{dy}{dx}$ of known function $y = f(x)$ may be obtained. The input to the differential is angular value y. Output u of the differential positions the ball carriage. The general integrator equation $Z = \int udx$ is fed into the differential as the other input. The action at the mechanical differential is $y - \int u\,dx = \frac{u}{k}$, where k is a fixed factor set to correct u in relation to differential inputs. Differentiation of this equation with respect to x gives, $\frac{dy}{dx} = u + \frac{1}{k}\frac{du}{dx}$. To obtain the correct answer, $\frac{1}{k}\frac{du}{dx}$ must equal zero. This condition is met when the ball carriage is stationary, since at this time $\frac{du}{dx}$ equals zero. In practice, however, it may not be possible to obtain zero motion of the ball carriage; but the error can be minimized by making the k value as large as possible without introducing too-high response of the ball carriage.

SPEED CONTROL

Figure 3 illustrates the application of the integrator as a variable-speed device. Output-roller speed may be varied from zero at disc center to a maximum of 2.4 times the input speed with 0.05% full-scale accuracy. Greater precision in speed control may be achieved when a differential is used with the integrator.

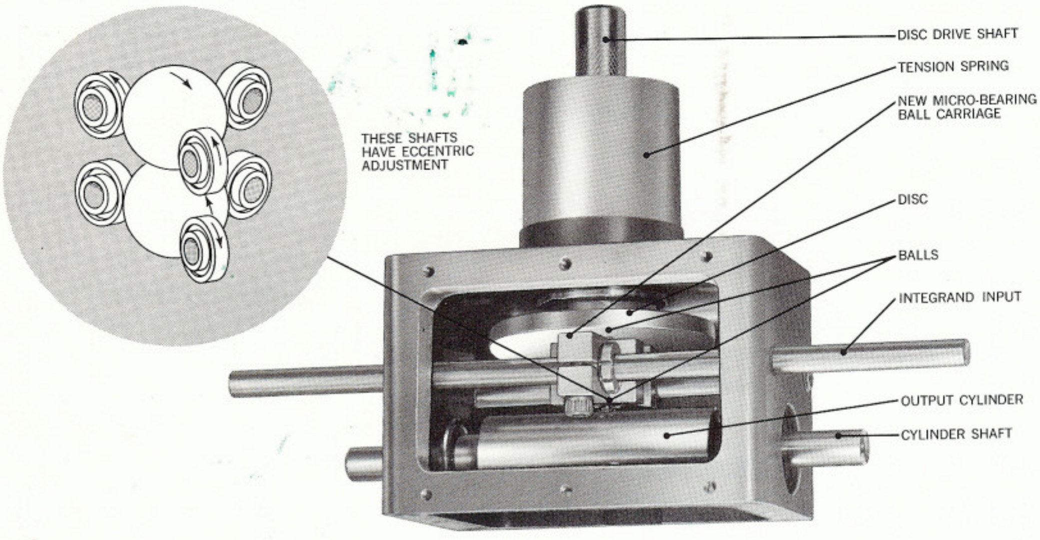
COMPUTING

The following equations may be represented with various ball and disc integrator arrangements:

- Output proportional to input $I = \int ydx = yx$
- Output proportional to input squared $I = \int cxdx = \frac{cx^2}{2}$
- Exponential integral $I = \int e^x dx = e^x$
- Product of two variables $I = \int \frac{Vdu + Udv}{2} = \frac{uv}{2}$
- Natural logarithm $I = \int \frac{1}{x} dx = \log_e x$
- Output proportional to reciprocal $I = \int \frac{1}{x^2} dx = \frac{1}{x}$
- Sine and cosine $\int \cos x\, dx = \sin x$, $\int \sin x\, dx = -\cos x$

INSTRUMENTATION AND PROCESS CONTROL

When used with suitable sensing elements, the integrator may be applied in a separate instrument or as a system element. Read-out devices commonly used for this type of application include mechanical counters, magnetic reed switches, and precision variable resistors.



GENERAL CHARACTERISTICS

DISC INPUT

Maximum breakaway torque . . . . . . 1.5 in. oz.
(no output load)
Maximum recommended input
speed . . . . 500 rpm

BALL-CARRIAGE INPUT

Maximum input force . . . . 2.0 oz.
Ball-carriage travel . . . 0.750 $^{+0.020}_{-0.000}$ in. either side of center (zero output)

OUTPUT

Maximum recommended load . . . . . 3.0 in. oz.
Reproducibility (% of full scale) . . . 0.01%
Accuracy (% of full scale) 0.05%
Output speed range . . . . 0 to 2.4 times input speed

LIFE EXPECTANCY

Conditions:
disc input . . . . 400 rpm
output load . . . . 3.0 in. oz.
Ball-carriage dynamic . . . . Minimum: 5000 hours
Expected: 10,000 hours

WEIGHT . . . . 17 oz.

SIZE . . . . $1\frac{7}{8}''$ wide, $2\frac{3}{4}''$ long, $3\frac{3}{4}''$ high

CONSTRUCTION

Case:	Aluminum alloy
Disc:	Tungsten carbide, precision-lapped to 1 microinch finish
Balls:	Instrument-grade steel
Carriage:	Aluminum alloy with adjustable microbearings
Roller:	Hardened 440C stainless steel
Bearings:	440C stainless steel

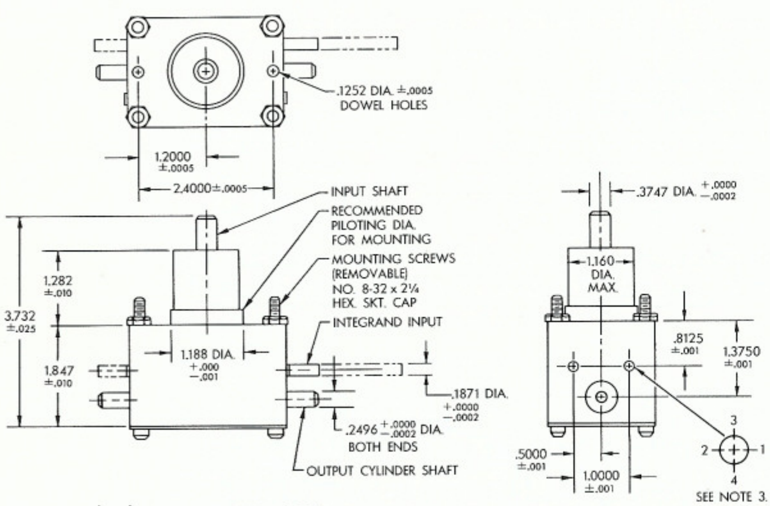
ORDERING INFORMATION

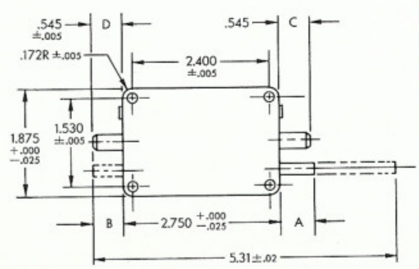
Specify:

1. Ball and Disc Integrator
2. Model series
   - Model series 20-1 (standard input shaft)
   - Model series 21 (rack input)
   - Model series 22-1 (standard input shaft threaded for 2.0 inches with 10-32 thread)
   - Model series 23 (lead-screw input, right-hand pitch)
3. For models 20-1, 21, 22-1, and 23, specify projection A or B. (See Note 1.)

DIM. A AND B	MODEL NO.
.531	20-1
.680	21
.531	22-1
.600	23

4. For Model 21, specify 64-, 72-, or 96-pitch rack Dimension from pitch line (20° pressure angle) Specify teeth position 1, 2, 3, or 4. (See Note 3.)
5. For Model 23, specify 8- or 40-pitch lead screw (right-hand pitch)
6. For all models, specify double-ended CD or single-ended output cylinder C or D. (See Note 2.)





NOTES:

1. "A" or "B" indicates projection of integrand input, rack, or lead screw with ball carriage at opposite internal stop.
2. "C" and "D" indicate projections of output cylinder.
3. 1, 2, 3, and 4 indicate rack teeth positions for Model 21 series. Rack may be at "A" or "B".

Design Guide

ANALOG COMPUTING MECHANISMS

... an explanation of the operational theory of ball-and-disc integrators and their applications in integration, differentiation, and speed changing.

BY GEORGE W. MICHALEC, SECTION HEAD, GENERAL PRECISION LABORATORY, PLEASANTVILLE, NEW YORK

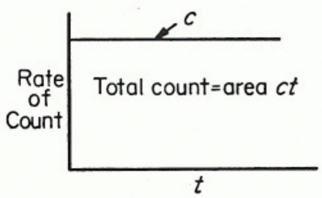


Fig. 9 — Constant-rate integration

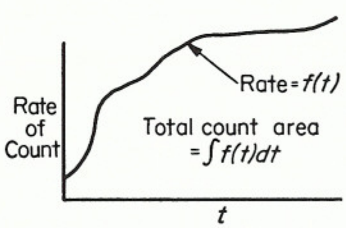


Fig. 10—Variable-rate integration

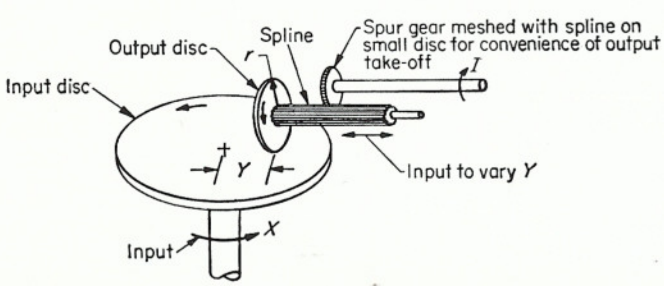


Fig. 11—Classical two-disc integrator

Integration and Differentiation

Performance of integration is an important computer capability because the accumulation of a variable is often required. Integration for a fixed rate of change for the variable is readily accomplished by a counting device driven by a motor at a fixed speed, Fig. 9. The integration procedure is much more difficult when the parameter to be integrated is a varying function, Fig. 10 .

Design of analog integration devices can be illustrated by an example: Integrate the distance traveled by an automobile. A simple way is to have one of the wheels drive a counter which gives a measure of wheel rotation or distance traveled. The wheel rolling on the road surface does the integration.

If the integration is to be done remotely, an analogous system must be provided. It could be a continuously driven, smooth-surfaced disc to represent the roadway, and a smaller contacting disc to represent the auto wheel, Fig. 11. Distance of the small driven disc from the center of the large driving disc, y, could be varied to represent the speed of the auto. Thus, output rotation of the small disc would be distance traveled, and the two-disc device would be an analog integrator. This is the classical two-disc integrator for which I = rotation of the small output disc, r = radius of the small output disc, x = rotation of large input disc, and y = radial position of the small disc on the large disc.

From the geometry of the rotating system,

$$2\pi x y = 2\pi r I$$

$$I = \frac{1}{r} x y$$

Since x, y, and I can be functions of a variable, such as time,

$$dI = \frac{1}{r} y\, dx$$

This equation states that any small change in x produces a corresponding change in the output I, but y/r times as much. Integration of the equation yields

$$I = \frac{1}{r} \int y\, dx \qquad (1)$$

Value $1/r$ is known as the integrator constant, a significant term in system synthesis. Thus, I is an integral of a function with one independent variable. Note that the direction of the output is reversed and requires a sign change when the center of the disc is traversed (to negative y) by the output disc. This represents a real mathematical integral. With proper control of x and y, this simple speed changer can become an integrator of lengthy and highly complex mathematical equations.

However, this unit is not a complete integrator in itself because it requires an accumulator, such as a counter, on its output. The rate of the small disc output is the derivative of the integral, or a velocity analog. In this example, it is the speed of the car.

Ball-and-Disc Integrator

The two-disc integrator has poor performance due to its low load characteristic. Output torque is proportional to the friction force between the two discs. Friction torque can be increased only by increasing the compressive load between them. Unfortunately, this would also increase the force necessary for sliding the small disc in order to vary the quantity y. Thus, the requirement for high torque output conflicts with the requirement of ease of shifting the take-off disc. Compromise results in low output torque which, in turn, necessitates a torque amplifier.

This limitation of the two-disc integrator is partly resolved by a ball-and-disc integrator. The output disc is replaced by two balls and a cylindrical roller. Thus, lower rolling-ball friction is substituted for high sliding-disc friction, Fig. 12. Use of much higher compressive forces results in significantly increased output torque.

Operating principle of the ball-and-disc integrator is identical to that of the two-disc integrator, and the same equations apply. Note that the integration constant, $1/r$, refers to the radius of the output roller. The ball radii are immaterial—they merely act as idlers.

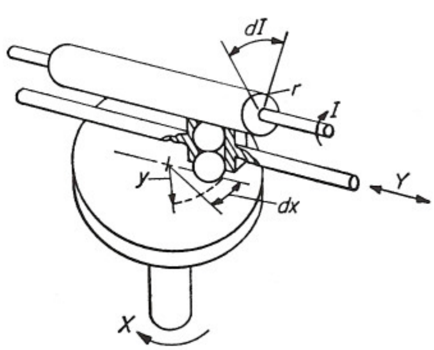


Fig. 12—Essentials of a ball-and-disc integrator

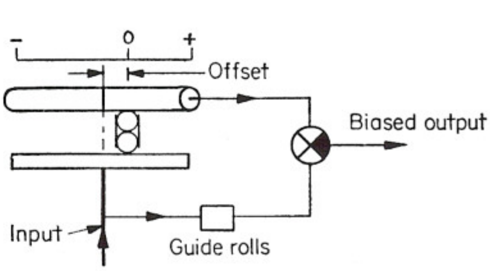


Fig. 13—Ball-and-disc integrator biased with a differential

Disc, balls, and roller are usually hardened steel alloys. This construction permits maximum compressive loads, and affords durability against wear and pitting. Carboloy discs are used in some of the most precise units. Size of the integrator is determined by the diameter of the disc. Thus, a two-inch disc is a "two-inch integrator." Sizes range from 1½ to 5 in. Several manufacturers offer commercial stock units.

Typical accuracy of precision units is 0.5 per cent up to 1 oz-in. load for units having a disc in the order of 1½ in. diameter.

The ball-and-disc integrator is a highly reliable unit. It can be designed for fairly rough service and for convenient incorporation into mechanical systems. It is relatively inexpensive.

The ball-and-disc integrator has two major faults: 1. Since it depends upon rolling friction, output torque is limited to several ounce-inches if high precision is desired. 2. In regular operation, the balls cause more slippage and wear at the center of the disc than elsewhere.

Wear at the center of the disc can be eliminated or reduced by several methods. Zero output can be biased to a radial value between the disc center and edge by displacing the ball carriage a slight amount from the center of the disc at the zero or null value of the ball carriage input. A differential and by-passed input can compensate for the offset, Fig. 13. Also, the balls can be induced to rotate with the disc at center position by means of controlled guide rolls, or the low friction of a ball-bearing sleeve.

Integrator Applications

The ball-and-disc integrator can be used for three fundamental operations: Integration, differentiation, and speed changing. In integration, the integral can be a function of one variable input (disc speed fixed, ball-carriage position varied), or two variable inputs (both disc speed and ball-carriage position varied).

Mathematical Integration: For mathematical analysis, Equation 1 is written without the integrator constant, $1/r$. Hence,

$$I = \int y dx \tag{2}$$

For this expression, Fig. 14 is the schematic representation. The ball carriage position (linear) is the function $y = f(x)$. Input to the disc is the quantity x, a circular measure of θ deg.

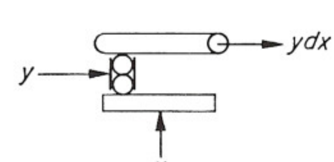


Fig. 14—Schematic essentials of a mechanical integrator

Since y must be linear in practice, rotary motion is converted to linear by a rack and gear, linkages, or the like. As the quantity x is varied, the disc input (x), and the ball carriage $y = f(x)$, are affected. The integral is the area under the curve. Adaptations of the integrator to varied conditions are shown in Computing with the Ball-and-Disc Integrator.

Differentiation: Mathematically, integration and differentiation are the reverse of each other. Therefore, one would expect that by reversing the integrator a differentiating mechanism would be obtained. However, to make the system feasible in a practical sense, a differential must be added. The result is a unit which theoretically does not give an exact mathematical solution, but, if applied properly, does give a usable answer. The schematic for this device is Fig. 15.

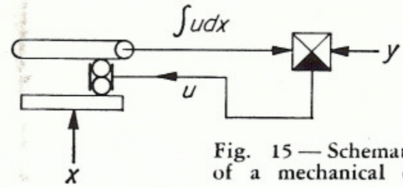


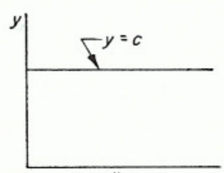
Fig. 15 — Schematic essentials of a mechanical differentiator

The integrator output is fed into a differential which has, as the other input, an angular value, y, which is a function of x. Output u of the differen-

Computing with the

Output Proportional to Input

Let x be varied at a constant rate, such as a fixed speed input. Then x, the angle input to the disc, increases as a straight-line function. The carriage position is constant ($y = c$) and the plot is a straight line parallel to the x axis.

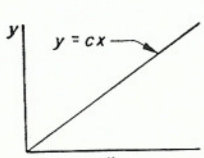


$$I = \int y\,dx = \int c\,dx = cx$$

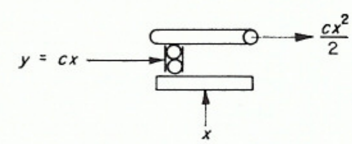
The area under the curve or integral, is the product xy or cx, since y is a constant. Constant c is the speed ratio of the integrator. For each ball-carriage position, there is a corresponding value of c.

Output Proportional to Input Squared

The integrator can be used to provide an output proportional to the input squared, if ball-carriage position y is varied linearly with x.



The integral or area under this curve is $cx^2/2$; hence, it is proportional to x^2. To produce this function, the integrator arrangement is

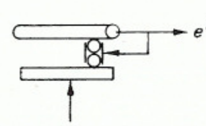


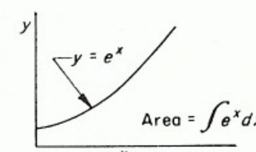
This arrangement is verified by substitution in the basic integrator equation.

$$I = \int y\,dx = \int cx\,dx = \frac{cx^2}{2}$$

Exponential Integral

Exponential output can be obtained by an integrator arranged for feed-back.





This arrangement is verified by assuming the answer and substituting in the basic equation,

$$I = \int y\,dx = \int e^x\,dx = e^x$$

Product of Two Variables

The product of two independent variables can be obtial positions the ball carriage. Positioning can be done by direct mechanical coupling, or, preferably, through power amplification. The differential inputs are arranged to be subtractive.

Suppose it is desired to obtain the derivative dy/dx of a known function $y = f(x)$. If the answer is assumed from the above schematic, the general integrator equation is

$$I = \int u\,dx$$

and the action at the mechanical differential is

$$y - \int u\,dx = \frac{u}{k}$$

where k is a fixed factor set to correct u in relation to the differential inputs. When this equation is differentiated with respect to x,

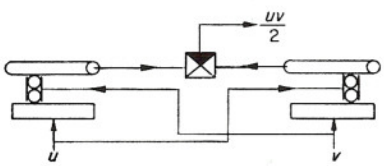
$$\frac{dy}{dx} = u + \frac{1}{k}\frac{du}{dx}$$

The assumed answer, u, is incorrect due to the second term, $(1/k)(du/dx)$. For the correct answer, this term must be zero, which can only be when $du/dx = 0$, that is, when the ball carriage is stationary. In practice, this does not occur because it would require infinite time. However, the error term can be minimized by setting k as large as possible without causing too high response of the ball-carriage motion.

Speed Changing: The standard ball-and-disc integrator can be a continuously variable speed changer. Precision construction enables adaptation to variable precision speed controls. However, in-

Ball-and-Disc Integrator

tained by using two integrators and a differential.



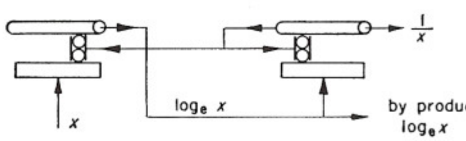
The arrangement is verified by substitution in the basic integration equation,

$$I = \int \frac{v\,du + u\,dv}{2} = \frac{uv}{2}$$

Note that the result illustrates the use of a mechanical differential.

Output Proportional to Reciprocal

Two integrators can be combined to yield a reciprocal output.



Again, proof is established by substitution of the assumed inputs and answers for y in the general equation. For the first integrator,

$$I = \int \frac{1}{x}\,dx = \log_e x$$

The output from the first integrator is the input to the second. Hence,

$$I = \int \frac{1}{x}\,d\log_e x$$

But,

$$\frac{d\log_e x}{dx} = \frac{1}{x}$$

Therefore,

$$\int \frac{1}{x}\,d\log_e x = \int \frac{1}{x^2}\,dx$$

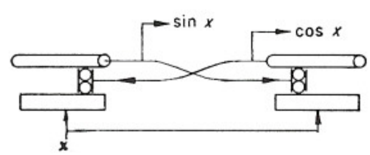
and the output of the second integrator is

$$I = \int \frac{1}{x^2}\,dx = \frac{1}{x}$$

A negative sign has no particular significance.

Sine and Cosine Functions

The trigonometric functions, sine and cosine, can be generated by means of two integrators:



The proofs are: For the output of the first integrator:

$$\int \cos x\,dx = \sin x$$

and for the output of the second integrator:

$$\int \sin x\,dx = -\cos x$$

herent low torque capacity is a limiting factor.

Ball-and-disc units can be used as simple speed changers, Fig. 16. In this arrangement, the ball-carriage position determines speed ratio, which can be varied from infinite (at disc center) to a low which is the reciprocal of the integrator constant. Because of the wide range, this arrangement is not very precise, particularly for high values.

A more sophisticated arrangement resulting in precision speed control can be achieved with the arrangement in Fig. 17.

In this scheme, the speed control affects only a portion of the output speed, due to the by-pass and added differential. Precise control of a base speed can thereby be obtained over a limited range. By proper choice of gear ratio, the ball carriage can be limited to any desired portion of the disc.

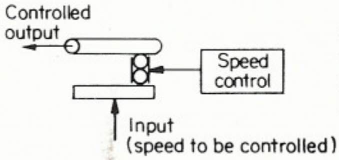


Fig. 16 — Ball-and-disc integrator arranged as a speed changer assembly

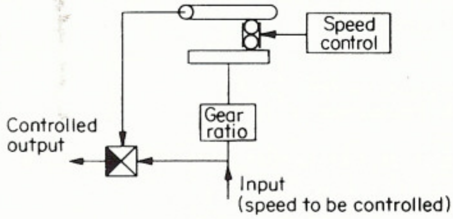


Fig. 17—Ball-and-disc integrator arranged as a precision speed control mechanism

Librascope's experience in the successful applications of mechanical computing devices—in many dissimilar situations—extends over thirty—five years.

This reservoir of knowledge is at your disposal. Please call upon our technical capabilities whenever we may be of service.

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